

A data structure for monomial ideals in signature Gröbner basis computations

Abstract:

Testing whether a monomial belongs to a monomial ideal is a fundamental operation in signature Gröbner basis computations [2][4], where such tests are performed repeatedly to detect redundant reductions and reductions to zero. We introduce monomial divisibility diagrams (MDDs) [5], a data structure for monomial ideals that supports incremental insertion of generators together with fast membership tests. MDDs arise from a canonical tree representation in which identical subtrees are maximally shared [3], yielding a compact directed acyclic graph representation.

We establish basic complexity bounds for membership and insertion, and study both empirically and theoretically the size of the MDDs arising in Gröbner basis computations. We integrate MDDs into the signature Gröbner basis implementation of the Julia package *AlgebraicSolving.jl* [1], and show that their use leads to substantial speed-ups.

References

- [1] Christian Eder et al. *AlgebraicSolving.jl*. URL: <https://github.com/algebraic-solving/AlgebraicSolving.jl>.
- [2] Jean Charles Faugère. “A new efficient algorithm for computing Gröbner bases without reduction to zero (F5)”. In: *Proceedings of the 2002 International Symposium on Symbolic and Algebraic Computation*. ISSAC '02. Lille, France: Association for Computing Machinery, 2002, pp. 75–83. ISBN: 1581134843. DOI: 10.1145/780506.780516. URL: <https://doi.org/10.1145/780506.780516>.
- [3] Jean-Christophe Filliâtre and Sylvain Conchon. “Type-safe modular hash-consing”. In: vol. 2006. Sept. 2006, pp. 12–19. DOI: 10.1145/1159876.1159880.
- [4] Pierre Lairez. “Axioms for a theory of signature bases”. In: *Journal of Symbolic Computation* 123 (July 2024), p. 102275. ISSN: 0747-7171. DOI: 10.1016/j.jsc.2023.102275. URL: <http://dx.doi.org/10.1016/j.jsc.2023.102275>.
- [5] Pierre Lairez, Rafael Mohr, and Théo Ternier. *A data structure for monomial ideals with applications to signature Gröbner bases*. 2026. arXiv: 2601.05026 [cs.SC]. URL: <https://arxiv.org/abs/2601.05026>.