Belyi map verification using certified path tracking

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A Belyi map is a three-point branched cover of the complex projective line and is characterized by its monodromy around each branch point. Using Algpath [2, 1], a certified path tracking software, we rigorously compute the monodromy triples and certify the correctness of the monodromy groups associated with the Belyi maps present on the LMFDB database [4, 5]. This demonstrates that recent advances in certified path tracking algorithms make them suitable for the verification step of the method of Klug, Musty, Schiavone, and Voight [3], step that is necessary to ensure that the resulting data is correct.

References

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