

# COMPUTING THE EIGENVALUES OF $QM$ -MATRICES AND $Q^{1,2}$ -MATRICES

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## ABSTRACT

$Q$ -matrices are matrices whose sums of principal minors of the same order are positive. A matrix is a  $QM$ -matrix if all its powers are  $Q$ -matrices. We will characterise, up-to size 5, the real  $QM$ -matrices and those real matrices  $A$ , up-to size 4, such that  $A$  and  $A^2$  are  $Q$ -matrices but not all eigenvalues of  $A$  have positive real part.

The study of the eigenvalues of these matrices (and its powers) brings many open questions. For example, until 2024 (see [3]), it was not known if the eigenvalues of a  $PM$ -matrix were necessarily positive (solving a longstanding conjecture raised in [2]). Or it is not known if the eigenvalues of a matrix  $A$  such that  $A$  and  $A^2$  are  $P$ -matrices necessarily have positive real parts (see [1]). Taking into account that a  $P$  (resp.  $PM$ ) matrix is a  $Q$  (resp.  $QM$ ) matrix, we will study these questions for  $Q$ -matrices and  $QM$ -matrices in order to find an answer for the original problems.

By using a mixture of symbolic-numeric techniques, we will show how to characterise the real  $QM$ -matrices up-to size 5 and to characterise those real matrices  $A$ ,  $4 \times 4$ , such that  $A$  and  $A^2$  are  $Q$ -matrices but not all eigenvalues of  $A$  have positive real part. Symbolic Computation is used to generate the characteristic polynomials of  $A^k$  (by using resultants) and to simplify the trigonometric expressions appearing as coefficients. Numerical Computation is used to graphically investigate the complicated systems of inequalities characterising, for example, a  $5 \times 5$   $QM$ -matrices (in this concrete case we will be dealing with an infinite number of inequalities).

## REFERENCES

- [1] D. Hershkowitz, and N. Keller. *Positivity of principal minors, sign symmetry and stability*. Linear Algebra and its Applications 364, 105–124 (2003).
- [2] D. Hershkowitz, and C.R. Johnson: *Spectra of matrices with  $P$ -matrix powers*. Linear Algebra and its Applications 80, 159–171 (1986).
- [3] S. Mondal, K.C. Sivakumar, and M.J. Tsatsomeros:  *$P$ -matrix powers*. Linear Algebra and its Applications 699, 355–366 (2024).

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