

# Classical Algebraic Geometry & Modern Computer Algebra: Innovative Software Design and its Applications

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Durham University

2024-07-25 ICMS 2024

# Agenda

- 1 What is this session about?
- 2 OSCAR and MaRDI
- 3 Why is this topic close to our hearts?
- 4 Organizational Remarks & Schedule

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**We are very excited to have all of you here, to listen to the fascinating talks and to discuss these topics we all care about deeply!**



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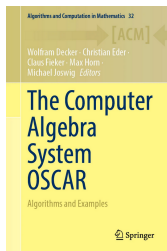
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- High-level integration of tools from different mathematical areas.
- Provides functions for groups, rings, fields, linear and commutative algebra, number theory, algebraic geometry, polyhedral geometry and much more.

# The OSCAR Book: <https://link.springer.com/book/9783031621260>



- Comprehensive guide and reference.
- Chapters demonstrate applications of OSCAR in a broad range of mathematical domains:
  - Group Theory,
  - Number Theory,
  - Polyhedral Geometry,
  - Algebraic Geometry and Commutative Algebra,
  - Various specialized topics (GIT fans, Invariant Theory, Matroids, Tropical Geometry, Toric Geometry, ...)

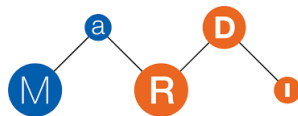
## Chapter preprints:

- Toric Geometry in OSCAR  
M. Bies & L. Kastner, <https://arxiv.org/abs/2303.08110>.
- Polyhedral Geometry in OSCAR  
T. Brysiewicz & M. Joswig, <https://arxiv.org/abs/2303.08110>.
- Elliptic fibrations on K3 surfaces  
S. Brandhorst & M. Zach, <https://arxiv.org/abs/2311.11766>.



## MaRDI

- **Mathematical Research Data Initiative**
- <https://www.mardi4nfdi.de/>




## Why is this topic close to our hearts?

We – **Martin Bies**, **Lars Kastner**, and **Matthias Zach** – are all contributing to the OSCAR project (funded by DFG as part of the SFB-TRR 195) as well as MaRDI (funded by DFG project ID 460135501).

OSCAR builds on various established, but rather specialized subsystems:

**GAP**



 polylake



**SINGULAR**

Images were generated using ChatGPT4o:  
OpenAI. (2024). ChatGPT (July 2024 version) [Large language model].  
Retrieved from <https://openai.com/chatgpt/>

- Flint
- Nemo/Antic
- Hecke
- Abstract Algebra

# OSCAR Tutorials: <https://www.oscar-system.org/tutorials/>



The OSCAR project

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## Tutorials

This page contains jupyter notebooks that demonstrate the functionality of the OSCAR project.

For each topic, you can decide to open a static version of the jupyter notebook, powered by [nbviewer](#). Alternatively, you can inspect the jupyter notebook directly on [github](#).

► [How to interact with a "live" version](#)

Click on one of the links below to filter notebooks (and re-click to disable filtering).

 <a href="#">FTheory Tools</a>	 <a href="#">Group Theory</a>	 <a href="#">Number Theory</a>	 <a href="#">Polyhedral Geometry</a>
 <a href="#">Commutative Algebra</a>	 <a href="#">Toric Geometry</a>		



**Polynomial Rings**  
 Author(s): John Abbott, Martin Bies, Luca Remke, Hans Schönemann  
 Last modified: July 12, 2024  
[nbviewer](#)  
[Github](#)



**Polyhedral Geometry**  
 Author(s): Martin Bies, Luca Remke  
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[nbviewer](#)  
[Github](#)

## We are always open to collaboration

- At this point there have been many OSCAR success stories from various fields in and adjacent to mathematics.
- If you need something implemented in OSCAR, please get in touch
  - Contact one of the many OSCAR developers here at ICMS
  - Talk to us at the panel discussion
  - Get in touch at <https://www.oscar-system.org/community/>

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- **Panel discussion:** “If I had the software to . . .” (15:00 to 15:30).
- Talk by Prof. Yang-Hui He concludes this session (16:00 to 17:00).

# Thank You!



**We appreciate your participation and look forward to the exciting talks and discussions ahead!**