

A Rainbow Clique Search Algorithm for BLT-Sets

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We consider the problem of classifying combinatorial-geometric structures by computer. A BLT-set is a set of $q + 1$ points in an orthogonal space $Q(4, q)$ defined over a finite field with q elements. The points satisfy a quadratic equation such as $x_0^2 + x_1x_2 + x_3x_4 = 0$ and there is a triple condition which says that no other point on the quadric is collinear to three of the points in the chosen set. The problem is that of classifying the possibilities of these sets up to equivalence under the orthogonal group.

After some reductions which involve the symmetry of the quadric, the problem can be phrased as a problem of finding rainbow cliques in suitable graphs [2]. This is a difficult problem in computer science [3] and quite a few CPU-cycles have been utilized to push the classification further. Several infinite families of BLT-sets are known but there is still a lot of example which are sporadic [4]. The present talk describes work to make the search more efficient and extend the classification to the next open case, which is when $q = 73$. Our computations are facilitated using the C++ package Orbiter [1] as well as some independent implementations utilizing multithreading.

References

- [1] Anton Betten. Classifying Discrete Objects with *Orbiter*. ACM Communications in Computer Algebra 01/2014; 47(3/4):183-186. DOI:10.1145/2576802.2576832
- [2] Anton Betten. Rainbow cliques and the classification of small BLT-sets. In *ISSAC 2013—Proceedings of the 38th International Symposium on Symbolic and Algebraic Computation*, pages 53–60. ACM, New York, 2013.
- [3] Coen Bron, Joep Kerbosch. Algorithm 457: finding all cliques of an undirected graph, Communications of the ACM, Volume 16 Issue 9, Sept. 1973, 575–577.
- [4] Maska Law. Flocks, generalised quadrangles and translation planes from BLT-sets. Thesis presented to the Department of Mathematics and Statistics, The University of Western Australia, March 2003.